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Equivalence-Principle-Based Active Metasurfaces

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Abstract

Considered a pillar of electromagnetic theory, the Huygens' principle draws a fundamental link between geometric optics and wave theory. This insight has led to the formulation of the equivalence principle which postulates that a field distribution within a region can be represented by fictitious equivalent sources on its boundary. While the equivalence principle remains a vital theoretical model, the advent of metasurfaces has provided a means of physically realizing such equivalent sources. To demonstrate the capabilities and potential of this technological development, this paper reviews recent work performed in the area of active Huygens' metasurfaces; composed of electric and magnetic radiating sources. These constituent sources can be used to replicate equivalence principle boundary conditions, enabling one to arbitrarily generate electromagnetic waves or control existing fields within a given region at will.

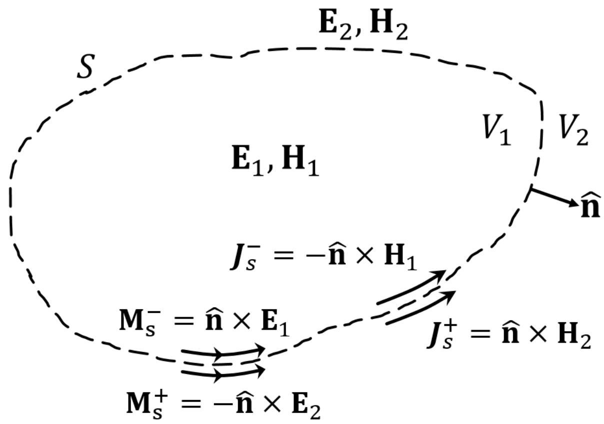


Figure 1. Equivalence principle model

1 Introduction

In 1678, Christiaan Huygens proposed that the behavior of light could be understood as a propagating wavefront. Moreover, every point along this wavefront could then be treated as a source of secondary spherical wavelets which constructively interfere in the direction of propagation [1]. This insight, now known as the Huygens' Principle, established a fundamental link between wave theory and geometrical optics. Initially, this spherical wavelets technique successfully elucidated the nature of electromagnetic wave propagation along with reflection and refraction. However, an explanation of diffraction effects was not achieved until 1818 when the principle was amended with an obliquity factor by Augustin-Jean Fresnel [2]. Later, the Huygens' principle was generalized into the equivalence principle by Love [3] and Schelkunoff [4]. Considered a foundational component of electromagnetic theory, the equivalence principle states that any electromagnetic field distribution within a region could be characterized as a product of fictitious equivalent sources placed along the region's boundary.

Recent technological developments have given rise to the development and proliferation of metasurfaces. Composed of sub-wavelength thin sheets of artificial material, metasurfaces can shape electomagnetic waves by imposing custom boundary conditions. This boundary condition dependent mechanism allows metasurfaces to be understood within the context of the Huygens'/equivalence principles. That is, when illuminated by an incident wave, the subwavelength elements within a metasurface act as secondary equivalent sources which interfere to generate a desired total field distribution. Along with providing a physical equivalent to theory, this relation could be used to simplify metasurface development and improve performance.

However, there exists more significant ramifications of this analogy. The ability to physically realize these, otherwise theoretical, equivalent sources permits the development of devices capable of arbitrarily generating and controlling waveforms. This work aims to demonstrate the feasibility and capabilities of such devices by introducing and reviewing research performed in active Huygens' metasurfaces. Instead of using passive elements, requiring excitation by an incident wave, these are comprised of an array of active, orthogonal electric and magnetic radiating sources. The equivalence principle can then be used to weight these sources to impose a desired regional field distribution.

This paper presents examples of active Huygens' metasurface designs. These are then implemented within a multitude of applications intended to showcase the ability to

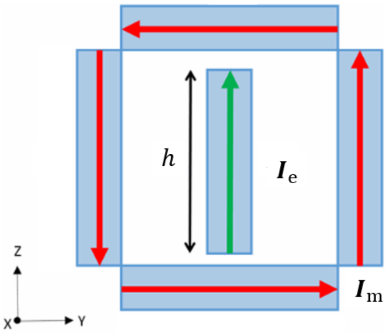


Figure 2. "Equivalence" source (adapted from: [5])

generate or control electromagnetic waves. These scenarios include the generation of naturally and non-naturally occurring modes within a closed conductive cavity, the recreation and phase-inversion of a scattered field to electromagnetically cloak an object, and the imposition of custom field distributions within a region in the presence of an existing incident wave.

2 Theory

2.1 Equivalence Principle

The equivalence principle [6] states that any electromagnetic field distribution within a region of space can be seen as the product of boundary-impressed radiating sources. Fig. 1 details a generalized model where an arbitrary closed surface S has been defined with an outward pointing normal of $\hat{\mathbf{n}}$. Here, $(\mathbf{E_1}, \mathbf{H_1})$ and $(\mathbf{E_2}, \mathbf{H_2})$ represent the fields inside and outside of the enclosed region. The equivalence principle can then be used to represent each field component as the product of an equivalent source. These sources are expressed as electric and magnetic surface currents impressed on their respective region facing sides of the boundary: $\mathbf{J_s^+} = \hat{\mathbf{n}} \times \mathbf{H_2}$, $\mathbf{M_s^+} = -\hat{\mathbf{n}} \times \mathbf{E_2}$, $\mathbf{J_s^-} = -\hat{\mathbf{n}} \times \mathbf{H_1}$, and $\mathbf{M_s^-} = \hat{\mathbf{n}} \times \mathbf{E_1}$. For a material boundary, individual equivalent electric and magnetic components sum to a total surface current distribution that embodies the field discontinuities between these regions.

$$\mathbf{J_s} = \mathbf{J_s^+} + \mathbf{J_s^-} = \hat{\mathbf{n}} \times (\mathbf{H_2} - \mathbf{H_1}) \tag{1}$$

$$\mathbf{M_s} = \mathbf{M_s^+} + \mathbf{M_s^-} = -\hat{\mathbf{n}} \times (\mathbf{E_2} - \mathbf{E_1}) \tag{2}$$

Initially serving as a theoretical tool, advances and proliferation of wireless technologies have pushed the equivalence principle into the realm of engineering and design. For example, the equivalence principle has been employed in the design, modeling, and evaluation of low profile radiators such as patch antennas [7]. More recently, the development of metasurfaces has provided a means of physically realizing these otherwise theoretical equivalent surface currents. This has opened the possibility of directly controlling the characteristics of a generated or existing waveform.

2.2 Equivalence Sources

Following from the equivalence principle, it is then evident that a desired E and H-field can be imposed within each region independently by inducing the appropriate equivalent surface currents along the boundary. Although defined as continuous properties, these surface currents can be physically approximated by a network of discrete electrically and magnetically responsive "equivalence sources". Fig. 2 illustrates a simple source design which consists of an electric dipole and a loop antenna (magnetic dipole). Here, an array of electric dipole antennas of length h and input current of $\mathbf{I_e}$ can be used to approximate an electric surface current of [9]:

$$\mathbf{J_s} = \mathbf{I_e}/s\,\hat{\mathbf{z}} \tag{3}$$

Likewise, an array of loop antennas with area A and fed with an electric current of $\mathbf{I_m}$ can approximate an orthogonal magnetic surface current:

$$\mathbf{M_s} = -\frac{j\omega\mu\mathbf{I_m}A}{s}\,\hat{\mathbf{x}} \tag{4}$$

Where s is the spacing between adjacent equivalence sources. It is also possible to develop simplified sources with hybrid responses such as the two-filament design in Fig. 3a. This element can simultaneously produce a set of orthogonal electric and magnetic dipole moments when the input currents of each filament are set as:

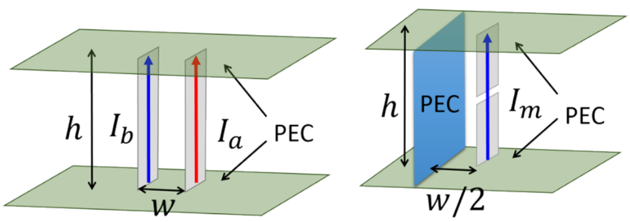
$$I_a = 0.5I_e + I_m \tag{5}$$

$$I_b = 0.5I_e - I_m \tag{6}$$

When situated in close proximity of a conductive surface, the two-filament equivalence source can be further simplified to the form pictured in Fig. 3b. In this case, the electric current in the dipole is mirrored across the conductive plane, creating a virtual loop element, while the electric surface current component is shorted out ($\mathbf{J_s} = 0$).

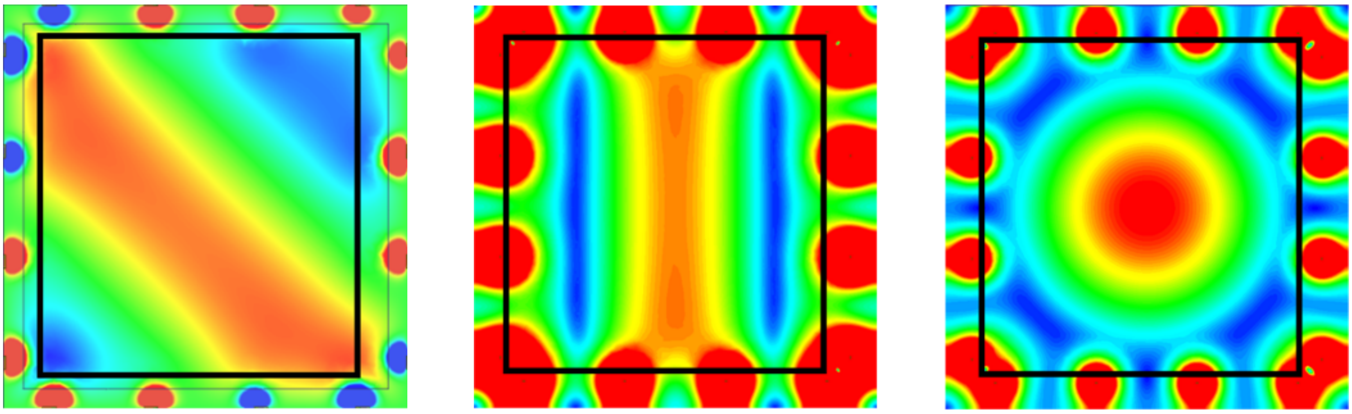
3 Applications/Field Synthesis

To illustrate the utility of equivalence based devices, the following sections detail active Huygens' metasurfaces created using the two-filament equivalence sources from Fig 3. Unlike their passive counterparts, active metasurfaces are not reliant on the existence of an incident field distribution, enabling a broader range of applications. The presented scenarios leverage the two primary capabilities of



(a) Conventional layout **(b)** Reduced layout

Figure 3. Two-filament equivalence source [8]



(a) Plane wave ($\theta = 45°$) **(b)** Standing wave **(c)** Bessel beam

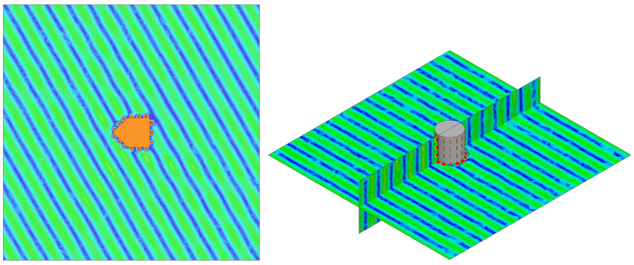
Figure 4. Generated E-fields within an enclosed cavity [10]

these active Huygens' metasurfaces: the generation of arbitrary waveforms and the manipulation of existing fields.

3.1 Control of Generated Waveforms

As previously specified, a characteristic advantage of active metasurfaces is their ability to operate in the absence of a pre-existing wave. This renders active Huygens' metasurfaces well suited for generating arbitrary field distributions at will, particularly within unoccupied regions. One such setup is detailed in Fig. 4 which plots the field distributions generated by equivalence sources lining a metallic, box-shaped cavity. As the cavity is conductive, $\mathbf{J_s} = 0$ and the sources can be implemented in reduced form (Fig. 3b). To simplify implementation and evaluation, a quasi-2D environment is created by limiting the height of the box to less than half a wavelength. This suppresses all non-TEM modes and allows a single layer of sources to be used.

A desired waveform can be generated by determining the fields along the boundary and using the equivalence principle to calculate the required surface currents and corresponding element weights. A simple case is shown in Fig. 4a which details the generation of a traveling plane wave at $\theta = 45°$ incidence within the cavity. More complex distributions, such as non-modal standing waves (Fig. 4b) or Bessel beams (Fig. 4c), can be similarly formed by calculating source weights directly from their respective electromagnetic profiles or by expressing the desired fields as a sum of plane waves [5], [10]. It is also notable that there are no theoretical limitations on the field distributions permitted; all of the modes presented in Fig. 4 do not naturally occur within the specified conductive cavity.



(a) Total E-field: 2D **(b)** Total E & H-fields: simplified 3D

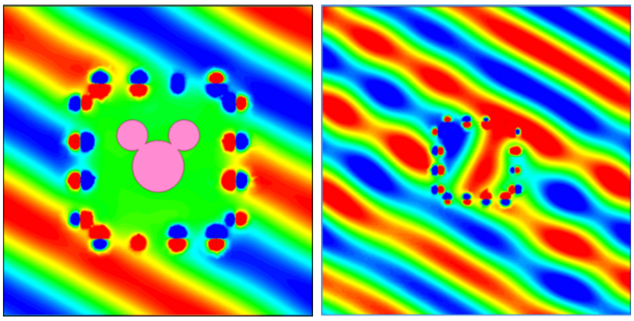
Figure 5. Cloaking a conductive cylinder [11]

3.2 Waveform Control in the Presence of an Incident Field

Although active Huygens' metasurfaces generate their own fields, they can also be employed to control and shape existing field distributions. Fig. 5a shows an exterior mounted Huygens' metasurface used to electromagnetically cloak a conducting cylinder from an axially (perpendicular) polarized incident plane wave. This is achieved using boundary conditions and the equivalence principle to determine a set of element weights to mimic the cylinder's scattered field [9], [12]. The generated field is then phased-inverted causing it to cancel out the naturally scattered fields [13], [11]. In addition to cloaking, the Huygens' metasurface generated fields can also be employed to alter the scattered field to mimic an object of a different shape, size, and material composition. While the metasurface shown here uses the simplified elements in Fig. 3b, the full two-filament layout is required if the target is non-conductive.

Fig. 5b demonstrates the feasibility of extending the active Huygens' metasurface cloak to a 3D environment. Since research on 3D active Huygens' surfaces is presently limited, this simulation only serves as an initial simplified proof of concept. Specifically, the incident wave remains axially polarized and constrained to a normal incident elevation while edge diffraction is suppressed by applying conductive boundary conditions tangential to the cylinder's top and bottom faces. Accordingly, a truly 3D implementation must accommodate for different polarizations, incident elevations, and edge diffraction.

Active Huygens' metasurfaces can also serve to “fence off" and control the fields within free-space regions [10]. Fig.



(a) Zero-field region **(b)** Internal wave region

Figure 6. Configurable field regions [10]

6a demonstrates this by employing a free-standing metasurface (Fig. 3a source configuration) to create a zero-field region within an incident plane-wave illuminated area. This is performed in a similar manner as the aforementioned cloak; the equivalent sources generate a field distribution within the designated region which cancels out the incident wave. Likewise, any object placed within the zero-field region is prevented from interacting with the incident wave and effectively cloaked.

Fig. 6b extends this concept further by using the active Huygens' metasurface to generate a secondary waveform to occupy the induced zero-field region. This configuration consequently operates as a free-space variation of the cavity in Fig. 4, allowing the metasurface enclosed region to theoretically acquire any desired field distribution. Furthermore, any object placed within this region will scatter the generated waveform rather than the exterior incident field.

4 Conclusion

The elements which comprise Huygens' metasurfaces can be envisioned as physical analogs of the fictitious sources described within the equivalence principle. Along with providing a physical counterpart to theory, the ability to manufacture these "equivalence sources" unlocks the possibility of real-world devices capable of arbitrarily generating or controlling electromagnetic waves.

To demonstrate the feasibility and utility of such devices, this paper presents the design and applications of active Huygens' metasurfaces, composed of arrays of electrically and magnetically responsive active radiating elements. In contrast to their passive counterparts, the use of active components eliminates operational dependence on the existence of pre-existing fields.

To demonstrate their capability to generate and control waveforms, the resultant active Huygens' metasurfaces are implemented in a variety of configurations. The first scenario uses an active Huygens' metasurface to line the walls of a conductive cavity. With proper configuration of the sources, any arbitrary field distribution can be generated within the cavity, including modes which do not naturally occur. A second configuration presents a Huygens' metasurface installed on a conductive cylinder illuminated by an incident plane wave. It is shown that this metasurface can be employed as an electromagnetic cloak by generating phase-inverted variations of the cylinder's scattered fields, canceling out their naturally occurring counterparts. The final configuration demonstrates the use of a free-standing Huygens' metasurface to control the fields within an arbitrarily defined free-space region, even in the presence of existing background radiation. These examples only represent a narrow scope of the possible applications of active Huygens' metasurfaces and much work still remains before practical technologies can be realized.

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